7.5: Conditional Probability and Dependence

Definition 1. Let A and B be events. Then the conditional probability of A with respect to B, or the probability of A given B, is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Example 1. If you roll a fair die twice and observe the numbers that face up, find the probability that the sum of the numbers is 8 given that the first number is 3.

Example 2. A survey of high school students found that, if a graduate went to college, there was a 40% chance that they would work at the same time. On the other hand, there was a 68% chance that a randomly selected graduate would go on to college. What is the probability that a graduate went to college and worked at the same time?

Example 3. An experiment consists of tossing two coins. The first coin is fair, whereas the second coin is twice as likely to land heads as tails. Find the probabilities of all possible outcomes.

Definition 2. The events A and B are independent if

$$P(A \cap B) = P(A)P(B).$$

Example 4. You roll two distinguishable dice and observe the numbers that are face up. Let A be the event that the first die is even and let B the event that both dice are even and let C be the event that the dice have the same parity (odds or evens). Determine which of the above events are independent of one another.

Question 1. According to the weather service, there is a 50% chance of rain in New York and a 30% chance of rain in Honolulu. Assuming that New York's weather is independent of Honolulu's, find the probability that it will rain in at least one of these cities.